Math 60 8.7 Absolute Value Equations and Inequalities (Day 2)

Objectives 3) Solve absolute value less than
less than or equal

$<, \le \Rightarrow$ AND

4) Solve absolute value greater than
greater than or equal

$>, \ge \Rightarrow$ OR

GOAL #1 Solve $|x| < 3$ or $|x| \le 3$

GOAL #2 Solve $|x| > 3$ or $|x| \ge 3$

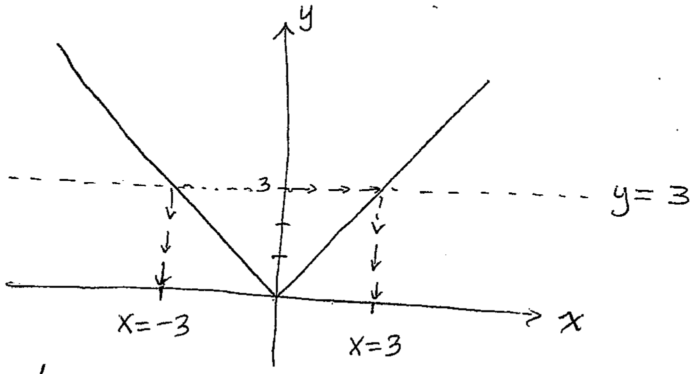
IMPORTANT: "Less than" problems are fundamentally different from greater than problems. Must know the difference.

LONG WAY

LESS THAN $<, \le$ PROBLEMS
Geometric Explanation #1

GOAL #1 $|x| < 3$

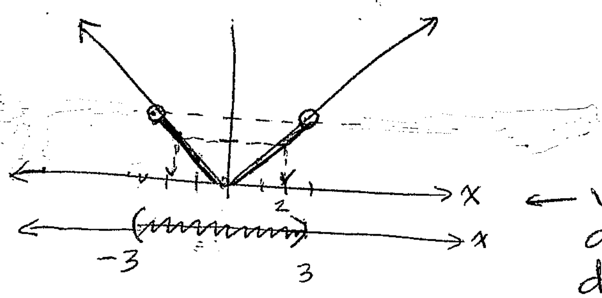
Graph $y = |x|$




Want $|x| < 3$
Means $y$ coordinates $< 3$

ex: if $x = 2$
$|2| < 3$ ✓

ex: if $x = -2$
$|-2| < 3$ ✓




← Want $x$-coordinates associated with the darkened part of graph.

Desired $x$ values

$-3 < x < 3$ (−3, 3) interval

Can also be written $x > -3$ and $x < 3$. INTERSECTION

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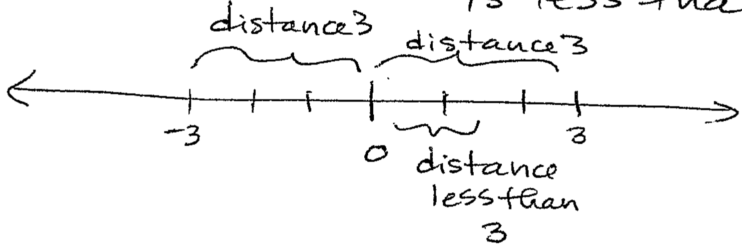
LONG WAY

## LESS THAN $<, \leq$ PROBLEMS
## Geometric Explanation #2

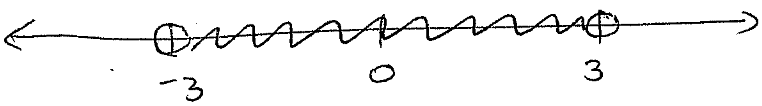
skip

$|x| < 3$ means "the values of $x$ so that

$|x - 0| < 3$ the distance from $x$ to 0 is less than 3"




Want all $x$ values




$(-3, 3)$ $\quad -3 < x < 3 \quad$ $x > -3$ and $x < 3$

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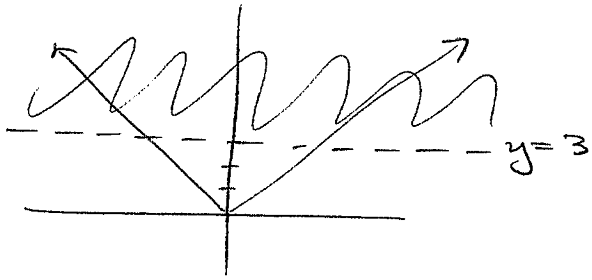
LONG WAY

**GREATER THAN >, ≥ PROBLEMS**
**Geometric Explanation #1**

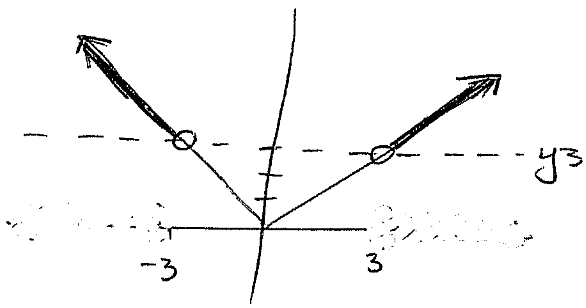
GOAL #2 $|x| > 3$

Graph $y = |x|$




← shaded area all y-coordinates greater than $y=3$

want $|x| > 3$
means y-coordinates $> 3$




want x-coordinates associated with the darkened part of graph

$(-\infty, -3) \cup (3, \infty)$

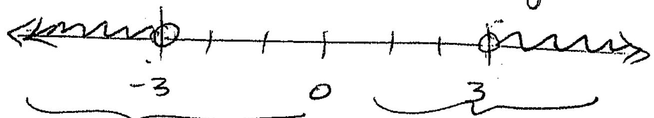
Desired $x$ values are in <u>two</u> locations

$x < -3$ <u>OR</u> $x > 3$ UNION

LONG WAY

**GREATER THAN >, ≥ PROBLEMS**
**Geometric Explanation #2** skip

$|x| > 3$ means "the values of $x$ so that the distance from $x$ to 0 is greater than 3"
$|x - 0| > 3$




distance greater than 3

distance greater than 3

$(-\infty, -3) \cup (3, \infty)$

$x < -3$ OR $x > 3$

UNION.

# Math 60 8.7 Day 2

## Algebraic Method - DO THIS!

- Isolate absolute value
- Make sure absolute value is on the left

YES → $|x-8| < 3$ NO → $3 > |x-8|$

$|x+7| \geq 5$ $5 \leq |x+7|$

- Write two inequalities. (a compound inequality)

1st inequality
- remove absolute value
- keep direction of inequality
- keep RHS unchanged

2nd inequality
- remove absolute value
* reverse direction of inequality
* opposite sign of RHS.

- Connect the two inequalities using logic

If $<$ or $\leq$, use AND/ $\cap$

If $>$ or $\geq$, use OR/ $\cup$

- Isolate the variable in 1st inequality
- " " " " 2nd "
- Graph the two on number lines
- Find the union (if $>$, $\geq$ problem)
or the intersection (if $<$, $\leq$ problem)

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Solve, graph the solution, and write the solution using interval notation.

(1) $\left|\frac{1}{2}x+1\right| \le 2$

check: absolute value is isolated ✓
absolute value is on left ✓

write 2 inequalities

$$\underbrace{\tfrac{1}{2}x+1}_{\text{remove abs value}} \quad \underbrace{\le}_{\text{keep}} \quad \underbrace{2}_{\text{keep}} \qquad \underbrace{\text{AND}/\cap}_{<,\le} \qquad \underbrace{\tfrac{1}{2}x+1}_{\text{remove abs value}} \quad \underbrace{\ge}_{\text{reverse}} \quad \underbrace{-2}_{\text{opposite}}$$

(*) (*) (*)

CAUTION:
(*) These three details are essential; forgetting one or more will substantially change your final answer ☹.

$$\tfrac{1}{2}x+1 \le 2 \qquad \cap/\text{AND} \qquad \tfrac{1}{2}x+1 \ge -2$$
$$-1 \quad -1 \qquad\qquad\qquad -1 \quad -1$$

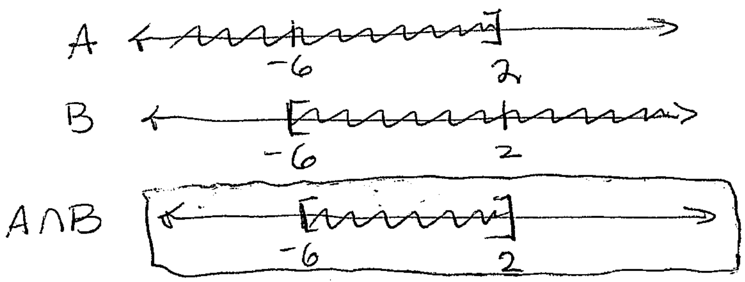
isolate x
- subtract 1
- mult by $\frac{2}{1}$ (same as $\div \frac{1}{2}$)

$$\tfrac{1}{2}x \le 1 \qquad\qquad \tfrac{1}{2}x \ge -3$$

$$2\cdot\tfrac{1}{2}x \le 2\cdot 1 \qquad\qquad 2\cdot\tfrac{1}{2}x \ge 2\cdot(-3)$$

$$\underbrace{x \le 2}_{A} \qquad \text{AND}/\cap \qquad \underbrace{x \ge -6}_{B}$$

Find $A\cap B$ using graphs.




Make sure −6 is on the left and +2 is on the right.

$A\cap B$ is where both A and B are shaded.

interval $\boxed{[-6,2]}$

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(2) $|3-2x|+1 < 6$

check: Is absolute value isolated? no. $A+1 \not< 6$ subtract 1

$|3-2x| < 5$

check: Is absolute value on left? yes. ✓

write two inequalities

$3-2x < 5$ AND/$\cap$ $3-2x > -5$

$-3 \quad -3$    $<, \leq$    $-3 \quad -3$

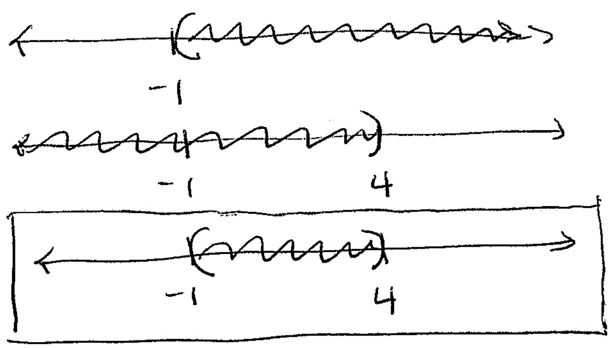
$\dfrac{-2x}{-2} < \dfrac{2}{-2}$      $\dfrac{-2x}{-2} > \dfrac{-8}{-2}$

$\underbrace{x > -1}_{A}$ AND/$\cap$ $\underbrace{x < 4}_{B}$

* Note *
÷ Neg means reverse inequality

Find $A \cap B$ by graphing

A
B
$A \cap B$



$\cap$/AND where both A and B are shaded

Interval $\boxed{(-1, 4)}$

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(3) $|2x+5| \geq 7$

check: Is absolute value isolated? ✓

Is absolute value on left? ✓

write two inequalities:

$2x+5 \geq 7$ OR/$\cup$ $2x+5 \leq -7$

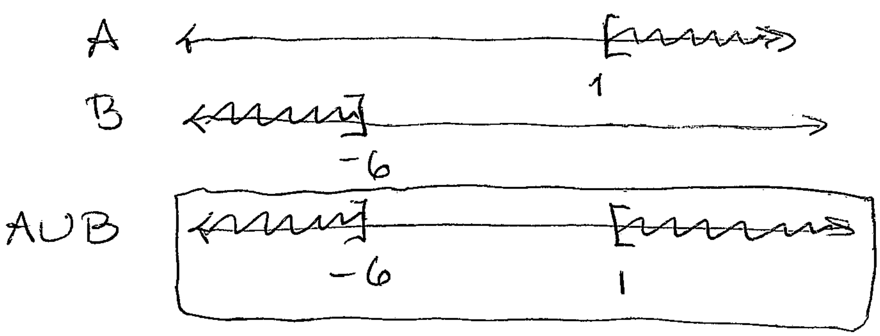
$>, \geq$

$\underline{-5} \quad \underline{-5}$ $\qquad$ $\underline{-5} \quad \underline{-5}$

$\frac{2x}{2} \geq \frac{2}{2}$ $\qquad$ $\frac{2x}{2} \leq \frac{-12}{2}$

$\underbrace{x \geq 1}_{A}$ OR/$\cup$ $\underbrace{x \leq -6}_{B}$

Find $A \cup B$ using graphs:




intervals $\boxed{(-\infty, -6] \cup [1, \infty)}$

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Solve and write in interval notation

4

$$-2|-3x-4|-5 < -6$$

$$\quad +5 \qquad +5$$

check: Isolated? No. Isolate absolute value

$$\frac{-2|-3x-4|}{-2} < \frac{-1}{-2}$$

continue to isolate the absolute value

CAUTION
$\div$ negative

$$|-3x-4| > \frac{1}{2}$$

Check: Left side? Yes.

greater than is OR, union

$$-3x-4 > \frac{1}{2} \quad \text{OR} \quad -3x-4 < -\frac{1}{2}$$

$$+4 \quad +4 \qquad\qquad +4 \quad +4$$

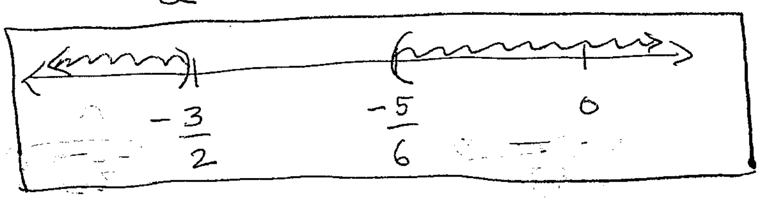
$$\left(-\frac{1}{3}\right)\cdot -3x > \frac{9}{2}\cdot\left(-\frac{1}{3}\right) \qquad \left(-\frac{1}{3}\right)\cdot -3x < \frac{5}{2}\cdot\left(-\frac{1}{3}\right)$$

CAUTION mult by neg

CAUTION mult by neg

$$\overbrace{x < \frac{-3}{2}}^{A} = -1.5 = \frac{-9}{6} \qquad \overbrace{x > \frac{-5}{6}}^{B} \approx -.8\overline{3}$$

find $A \cup B$




$$\boxed{\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{5}{6}, \infty\right)}$$

Notice:
$-\frac{9}{6} < -\frac{5}{6}$ OR
$-1.5 < -.8\overline{3}$
means
$\frac{-3}{2}$ left of $\frac{-5}{6}$

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(5) The inequality $|x-98.6| \geq 1.5$ represents a human body temperature, measured in °F that is considered "unhealthy." Solve the inequality and interpret results.

$$|x-98.6| \geq 1.5$$

greater than means OR/UNION

$$\begin{array}{lcl} x-98.6 \geq 1.5 & \text{OR} & x-98.6 \leq -1.5 \\ \underline{+98.6 \quad +98.6} & & \underline{+98.6 \quad +98.6} \\ x \geq 100.1 & \text{OR} & x \leq 97.1 \end{array}$$

$$\boxed{(-\infty, 97.1) \cup (100.1, \infty)}$$

Temperatures above 100.1 are fevers. (unhealthy)
Temperatures below 97.1 are unhealthy.

(6) The inequality

$$|p-67| \leq 4$$

results from the statements "67% of Americans experienced hardship due to gasoline prices. The margin of error of this study is 4%."
Solve the inequality and interpret the results.

$$\begin{array}{lcl} p-67 \leq 4 & \text{AND} & p-67 \geq -4 \\ \underline{+67 \quad +67} & & +67 \quad +67 \\ p \leq 71 & \text{AND} & p \geq 63 \end{array}$$

$(63, 71)$

Between 63% and 71% of Americans experienced hardship due to gasoline prices.

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(1) $3|z| + 8 > 2$

<u>step 0</u>: Isolate absolute value on LHS.

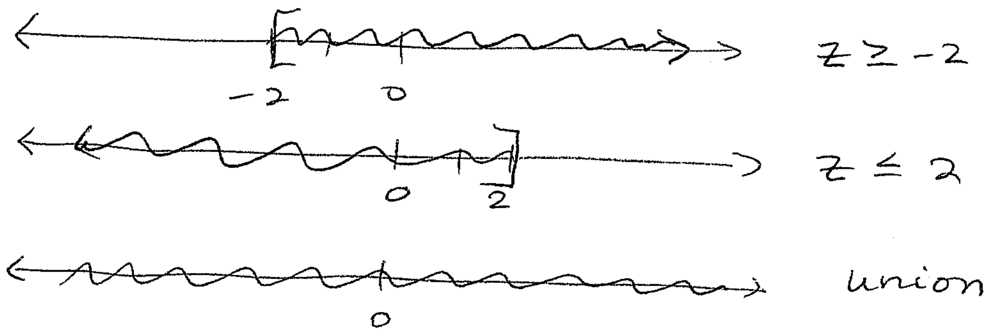
$$\frac{3|z|}{3} \geq \frac{-6}{3}$$

$$|z| \geq -2$$

<u>step 1</u>: Greater than means OR/UNION

$z \geq -2$ OR $z \leq -(-2)$

$z \leq 2$




$$\boxed{(-\infty, \infty)}$$

Math 60 8.7 Day 2

(8) $|3-5x| > \underbrace{|-7|}$

notice: No variable!

$|-7| = 7$

$|3-5x| > 7$

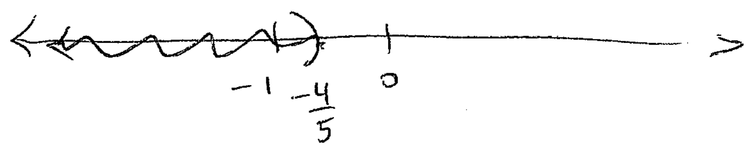
$3-5x > 7$ or $3-5x < -7$

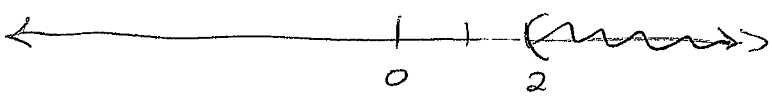
$-3 \qquad -3$ $\qquad\qquad$ $-3 \qquad -3$

$\frac{-5x}{-5} > \frac{4}{-5}$ $\qquad\qquad$ $\frac{-5x}{-5} < \frac{-10}{-5}$

* div by neg * $\qquad\qquad$ * div by neg *

$x < -\frac{4}{5}$ OR $x > 2$




$$\boxed{(-\infty, -\tfrac{4}{5}) \cup (2, \infty)}$$